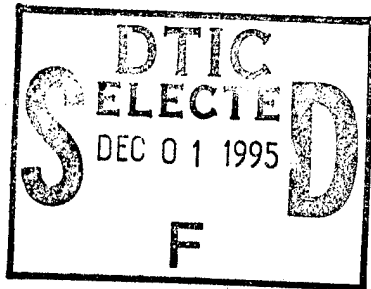# SIMULATION METHODOLOGY

by

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and

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FINAL REPORT

August 1994

Prepared under the Auspices

of

US Army Research Contract

DAAL03-91-G-0101

DEPARTMENT OF OPERATIONS RESEARCH
STANFORD UNIVERSITY
STANFORD, CA 94305

1. INTRODUCTION

This contract covered the period April 15, 1991 through April 14, 1994. The participating personnel included two faculty members and nine Ph.D. students. The research carried out under this contract involved both work on simulation methodology and stochastic models/processes. The simulation work involved the use of simulation to carry out stochastic system optimization, a variety of variance reduction ideas to improve simulation efficiency, and methods for improving output analysis for a single stochastic process. Stochastic models studied involved a network of water reservoirs, the general single-server queue, and a financial model for trading securities. Stochastic processes studied were cumulative processes, counting processes, and sampling of a general stochastic press.

2. DESCRIPTION OF RESEARCH

Our work on simulation methodology begins with the formulation of a model as a stochastic process. We then use the structure of the stochastic process to develop methods for analyzing the output of the simulation. Diagram 1 is a block diagram which details our approach to system simulation. The performance evaluation and reliability analysis of complex engineering systems requires an ability to analyze mathematical models of these systems. Large-scale stochastic models are required to handle various uncertainties present in these systems. Unfortunately, the complexity of most stochastic models of real systems is well beyond our ability to apply classical mathematical analysis. Computer simulation of stochastic systems has become one of the principal alternatives to classical analysis. The main thrust of our research is improving the efficiency of simulation methods and extending their applicability to a wider class of stochastic systems. We also carry out research on a variety of stochastic models with the aim of obtaining closed form solutions or approximations.

The recent topics being pursued in this project fall into four main categories: importance sampling for rare events, diffusion approximations for complex stochastic models, Monte Carlo evaluation of integrals, and stochastic optimization. In all these categories our approach is to use the detailed stochastic structure of the system under study to obtain more refined results than could be achieved using a "black box" approach.

An increasing number of capital-intensive technologies are intended to offer very high standards of reliability and/or availability (e.g., air traffic control systems, communications networks, airline reservation systems, manufacturing lines). Rare event simulation arises when studying these highly reliable systems that are designed to rarely fail. Naive simulation of such systems may require enormous amounts of computer time. Our approach is to use importance sampling in which the system is forced to fail more frequently along a path suggested by the mathematical theory of large deviations. The final computation must be adjusted by the likelihood ratio in order to estimate the original system reliability. Steady-state performance measures for highly dependable systems (such as fault-tolerant computer systems) are estimated by simulations involving rare events. Earlier work dealt with Markovian systems that could be formulated as regenerative process. Our current work assumes that repair and failure times are non-exponential and this leads to non-Markovian systems for which the regenerative structure is lost. A new approach has been developed using importance sampling applied to a sequence of cycles which are no longer independent, identically distributed as in the regenerative case. Experimental results show that the method is effective in practice. Gradient estimation is required when optimization of steady-state performance measure is being carried out with respect to a vector of design parameters. Conditions have been developed under which the likelihood ratio methods and infinitesimal perturbation analysis techniques are valid.

Complex stochastic models often defy efforts to obtain closed form solutions. Sometimes it is possible to construct a sequence of such models indexed by a physical parameter and to prove a limit theorem for the sequence as the parameter approaches a limit. The idea is to then use the limit process so obtained to compute approximations for the original node. This approach has been used in two recent papers devoted to complex repair systems and to networks of water reservoirs. In both cases the limit process is a multi-dimensional Gaussian diffusion process. For the complex repair system a technique was developed to analytically gauge the goodness of the diffusion approximation. In most cases the approximation was quite good.

For a network of continuous time reservoirs with power law release rules, convergence of a system of processes to a multi-dimensional Gaussian diffusion process is established. Simulations of the original watershed model were carried out to validate the use of the diffusion approximations. Conditions are developed for the steady-state waiting time tail probabilities in a single-serve queue to be asymptotically exponential. These conditions cover the case of multiple independent sources which arise in the design and control of emerging high-speed communication networks.

Monte Carlo simulation strategies are commonly used to numerically evaluate high-dimensional multiple integrals. The associated estimators have convergence rates which are typically $n^{-1/2}$, where n is the number of function evaluations made, and are independent of the dimension d of the integral. Standard deterministic integration schemes often have rates of convergence of the order $n^{-1/d}$. Monte Carlo methods construct estimators involving sample means of function evaluations computed at some randomly chosen points. All information about the random locations at which the function evaluation were computed is discarded. We have shown in recent research that using this location information for one-dimensional integration problems can improve the rate of convergence from $n^{-1/2}$ to n^{-2}. This suggests that high-dimensional Monte Carlo sampling algorithms could also take advantage of the additional function evaluation location information.

A recent trend in simulation research has been to use simulation as a vehicle for system optimization. Suppose a system can be described in terms of a finite set of parameters. The system is sufficiently complex that its performance can only be evaluated via simulation. In order to carry out an optimization of the system we need to estimated the gradient of the performance measure as well as its value. Two methods that have been proposed for gradient estimation are the likelihood ration and perturbation analysis techniques. Recent research has developed conditions under which these two techniques are valid.

Stochastic differential equations are used to model many types of economic and physical systems. Often these equations cannot be solved analytically and simulation is used to estimate the solution. When simulation is used, the (continuous) time variable is discretized. When a fixed budget for computer time is available, the simulator must trade off the size of the discrete time step and the number of replications that can be made. This trade-off is resolved for both first-order discretization schemes (such as Euler) and second-order and higher schemes (such as those of Milshtein and Talay).
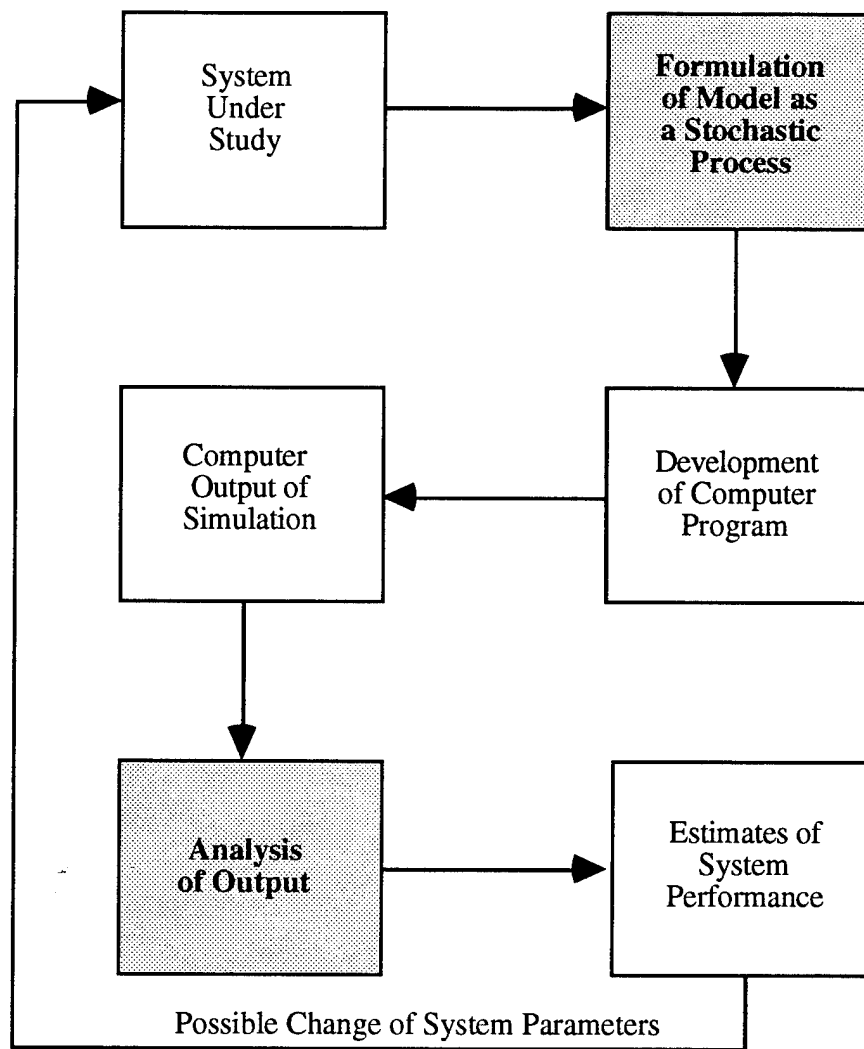
Diagram 1. Technical Approach to System Simulation*
*Our emphasis is on the shaded boxes

3. SCIENTIFIC PERSONNEL

The following scientific personnel were supported on this contract.

Faculty supported:

Peter W. Glynn, Associate Professor and co-Principal Investigator

Donald L. Iglehart, Professor and co-Principal Investigator

Ph.D Students Supported and their Dissertation Titles:

Nathaniel Chan, earned M.S. and Ph.D. in Operations Research.
Optimal Hydraulic Aquifier Management with Reliability Constraints

Diane Erdmann, earned M.S. in Statistics and Ph.D. in Operations Research.
Complexity Measures for Testing Binary Keystreams

Sandeep Juneja, earned Ph.D. in Operations Research.
Efficient Rare Event Simulation of Stochastic Systems

Craig Kollman, earned Ph.D. in Statistics.
Rare Event Simulation in Radiation Transport

Tava Lennon, Ph.D. earned Ph.D. in Operations Research.
Response-Time Approximations for Multi-Server Polling Models, with Manufacturing Applications

Wing Wah Loh, Ph.D. candidate in Operations Research.
Methods of Control Variates for Discrete Event Simulation

Eugene Wong, Ph.D. candidate in Operations Research.

Tzu-Hui Yang, Ph.D. candidate in Operations Research.
Efficient Simulation Techniques with Application to Performance Evaluation of ATM Switches

Tim Zajic, earned Ph.D. in Operations Research.
Large Deviations for Sample Path Processes and Applications

Honors

Peter W. Glynn and Ben Fox were awarded the 1993 TIMS College on Simulation prize for their paper "Discrete-time Conversion for SImulating Finite-Horizon Markov Processes" *SIAM J. Appl. Math.* 50 (1990) 1457-1473.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE August 1994	3. REPORT TYPE AND DATES COVERED Final Report

4. TITLE AND SUBTITLE

Simulation Methodology

5. FUNDING NUMBERS

6. AUTHOR(S)

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Operations Research
Stanford University
Stanford, CA 94305-4022

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This contract covered the period April 15, 1991 through April 14, 1994. The participating personnel included two faculty members and nine Ph.D. students. The research carried out under this contract involved both work on simulation methodology and stochastic models/processes. The simulation work involved the use of simulation to carry out stochastic system optimization, a variety of variance reduction ideas to improve simulation efficiency, and methods for improving output analysis for a single stochastic process. Stochastic models studied involved a network of water reservoirs, the general single-server queue, and a financial model for trading securities. Stochastic processes studied were cumulative processes, counting process, and sampling of a general stochastic press.

14. SUBJECT TERMS

15. NUMBER OF PAGES
7

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102